On statistical graph and pointwise convergence of sequences of set-valued functions defined on intuitionistic fuzzy normed spaces

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Introduction

Fuzzy set theory, introduced by Zadeh (1965), and its generalization, intuitionistic fuzzy set theory by Atanassov (1986), have significantly advanced pure and applied mathematics. This study extends the concepts of statistical graph and pointwise convergence, originally developed for set-valued functions in metric spaces by Aubin and Frankowska (1990), to intuitionistic fuzzy normed spaces (IFNS). We introduce statistical graphical, and pointwise limits for sequences of set-valued functions defined from one IFNS to another, exploring their convergence properties and relationships, supported by illustrative examples.

Methodology

We define statistical pointwise and statistical graphical limits using natural density and IFNS topology, as per Hosseini et al. (2007). The set-valued functions map from an IFNS $\left(X, I_{\varphi_1,\psi_1}, T\right)$ to another INFS $\left(X, I_{\varphi_2,\psi_2}, T\right)$. Key definitions include statistical pointwise upper/lower limits $st_p-\mathcal{F}_n^u$ / $st_p-\mathcal{F}_n^l$ and graphical limits $st_g-\mathcal{F}_n^u$ / $st_g-\mathcal{F}_n^l$. We generally investigate convergences using the lattice $\mathcal{L}^*$ and t-norm T with examples constructed in $\mathbb{R}$ and $\mathbb{R}^2$. Theorems are proved to establish relationships between convergences, incorporating U-intuitionistic fuzzy equi-continuity.

Results

The study establishes that pointwise and graph convergent sequences are statistically convergent, but the converse is not true (Example 3.1). Statistical boundedness does not guarantee convergence (Example 3.2). Theorems 3.7 and 3.8 show that under U-IFUEC conditions, statistical pointwise and graph convergences are equivalent. Example 3.8 highlights the necessity of statistical compactness.

Discussion

The results generalize topological convergence to the intuitionistic fuzzy context, revealing nuanced relationships between pointwise and graph convergences. The U-IFEC property is critical for equivalence, though limitations arise without compactness. These findings enhance set-valued analysis in fuzzy environments, with applications in optimization and control theory.

Conclusion

This work introduces and clarifies statistical convergence in IFNS, offering new definitions and theorems. Future work will explore applications in variational analysis and intuitionistic fuzzy optimization. The study provides a robust framework for analyzing set-valued functions in fuzzy settings.

References

- Hosseini SB, Oregan D, Sadati R. Some results on intuitionistic fuzzy spaces. Iran. J. Fuzzy Syst. 4(1): 53–64, 2007.
- Attouch H. Variational convergence for functions and operators. Pitman, London, 1984.
- Prete ID, Iorio MD, Holá L. Graph convergence of set-valued maps and its relationship to other convergences. J Appl Anal 6(2):213–226, 2000.
- J.P. Aubin, H. Frankowska. Set-Valued Analysis, Birkhäuser Boston Inc., Boston, MA, 1990.

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