Formulation of Total Shortening Distance for Adding Close Relations between One Member and Every Other Member of the Same Level in a Complete Binary Linking Pin Organization Structure

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Introduction

A linking pin organization is a structure in which relations between members of the same section are added to a pyramid organization where there exist only relations between each superior and his direct subordinates.

This study proposes a model of adding relations between one member and every other member of the same level N in a complete binary linking pin organization structure where edges are added between every pair of nodes which have the same parent in a complete binary tree of height H. When the lengths of adding edges are 0.5 while those of edges of the complete binary linking pin organization structure are 1, the total shortening distance which is the sum of shortening lengths of shortest paths between every pair of all nodes by adding edges is formulated for the purpose of revealing an optimal additional relation level N^*.

Methods

Let $S_H(N)$ denote the total shortening distance, when we add edges between one node and every other node of the same depth N (N = 1, 2, ..., H) in a complete binary linking pin organization structure of height H(H = 1, 2, ...). The lengths of adding edges are 0.5 while those of edges of the complete binary linking pin organization structure are 1.

The total shortening distance $S_H(N)$ can be formulated by adding up the following three sums of shortening distances: (i) the sum of shortening distances between every pair of nodes whose depths are equal to or more than N, (ii) the sum of shortening distances between every pair of nodes whose depths are less than N and those whose depths are equal to or more than N and (iii) the sum of shortening distances between every pair of nodes whose depths are less than N.

Formulation

The sum of shortening distances between every pair of nodes whose depths are equal to or more than N is given by

$$A_H(N) = 0.5\{M(H-N)\}^2(2^N-1) + \{M(H-N)\}^2 2^N \sum_{i=1}^{N-1} i2^i, \quad (1)$$

where $M(h)$ denotes the number of nodes of a complete binary tree of height h(h = 0, 1, 2, ...). The sum of shortening distances between every pair of nodes whose depths are less than N and those whose depths are equal to or more than N is given by

$$B_H(N) = 0.5M(H-N)\{M(N-1)-N\} + 0.5M(H-N)\sum_{i=1}^{N-1} i2^i + M(H-N)2^{N+1}\sum_{i=1}^{N-2} i(N-i-1)2^i, \quad (2)$$

and the sum of shortening distances between every pair of nodes whose depths are less than N is given by

$$C(N) = 0.5\sum_{i=1}^{N-2} i(N-i-1)2^i + 2^N \sum_{i=1}^{N-3}\sum_{j=1}^{i} j(i-j+1)2^j, \quad (3)$$

where we define $\sum_{i=1}^{0} \bullet = 0$ and $\sum_{i=1}^{-1} \bullet = 0$.

From Eqs.(1), (2) and (3), the total shortening distance $S_H(N)$ is given by

$$S_H(N) = A_H(N) + B_H(N) + C(N). \quad (4)$$

Conclusions

This study considered obtaining an optimal depth N^* of which edges between one node and every other node in a complete binary linking pin organization structure of height H are added. In this study the lengths of adding edges are 0.5 while those of edges of the complete binary linking pin organization structure are 1. The total shortening distance which is the sum of shortening lengths of shortest paths between every pair of all nodes by adding edges was formulated.