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Stability and Second-Order Lateral Stiffness of Embedded Piles with Generalized End-Boundary Conditions on Non-Homogeneous Soil

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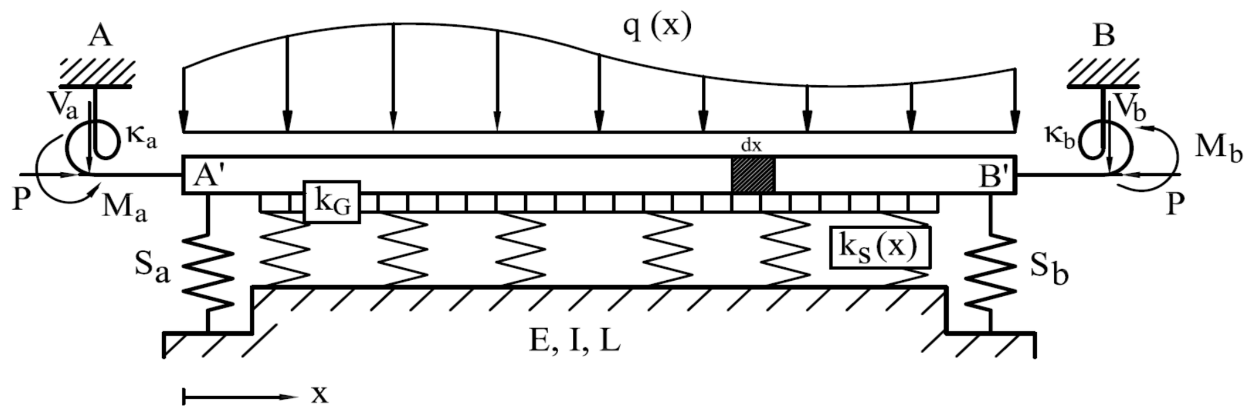
Introduction: A new, simplified analytical method to conduct elastic stability and second-order lateral stiffness analysis of piles with generalized end-boundary conditions on non-homogeneous elastic soil is derived in a classical manner and presented in detail. The influence of the modulus of subgrade reaction, degrees of non-homogeneity, and intermediate end-boundary conditions on the pile response are investigated via a parametric study. The proposed solution can be employed to perform either lateral deformation or elastic buckling analysis.

Structural Model: The pile is connected to ends *A* and *B* by semi-rigid connections and linear transverse springs with stiffness k_a and S_a, and k_b and S_b, respectively. The pile has a stiffness *EI*, Length *L*, and is embedded in a two-parameter elastic soil. The modulus of subgrade reaction k_s varies in a linear fashion following the expression $k_s(x) = k_o$+cx.



Governing Differential Equation (GDE):

$$\frac{d^4\bar{y}}{d\bar{x}^4} + F\frac{d^2\bar{y}}{d\bar{x}^2} + \Lambda(\bar{x})\bar{y} = \Omega(\bar{x})$$

where $F = (L^2/EI)(P - k_G)$, $\Lambda(\bar{x}) = (L^4/EI)(k_0 + cL\bar{x})$, and $\Omega(\bar{x}) = (L^3/EI)(a_0 + a_1L\bar{x} + a_2(L\bar{x})^2)$

Boundary Conditions:

At $\bar{x} = 0$

$$M_a - \frac{3EI\rho_a}{(1-\rho_a)L}\frac{d\bar{y}}{d\bar{x}} + \frac{EI}{L}\frac{d^2\bar{y}}{d\bar{x}^2} = 0 \qquad V_a - S_aL\bar{y} - (P - k_G)\frac{d\bar{y}}{d\bar{x}} - \frac{EI}{L^2}\frac{d^3\bar{y}}{d\bar{x}^3} = 0$$

At $\bar{x} = 1$

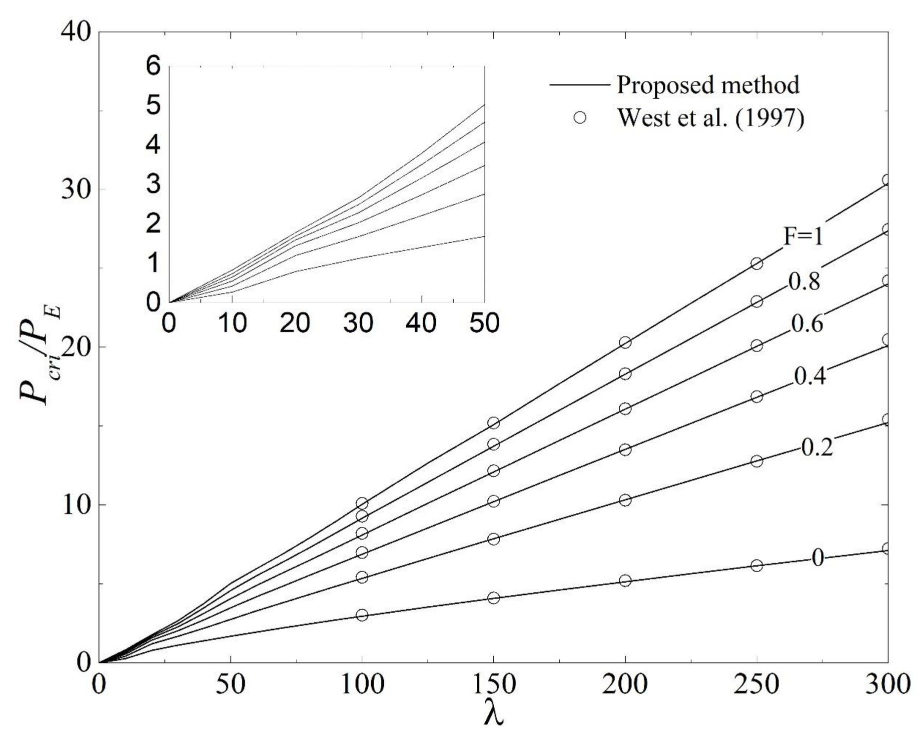
$$M_b - \frac{3EI\rho_b}{(1-\rho_b)L}\frac{d\bar{y}}{d\bar{x}} - \frac{EI}{L}\frac{d^2\bar{y}}{d\bar{x}^2} = 0 \qquad V_b - S_bL\bar{y} + (P + k_G)\frac{d\bar{y}}{d\bar{x}} + \frac{EI}{L^2}\frac{d^3\bar{y}}{d\bar{x}^3} = 0$$

Solution: The Differential Transformation Method (DTM) was used to find the solution to the GDE. This complex problem is reduced to solve a polynomial function, where the coefficients of the series are found from a recursive equation obtained from the GDE.

$$y(\bar{x}) = \bar{Y}(0) + \bar{Y}(1)\xi + \bar{Y}(2)\xi^2 + \bar{Y}(3)\xi^3 \ldots + \bar{Y}(m)\xi^m = \sum_{k=0}^{\infty}\bar{Y}(k)\xi^k$$

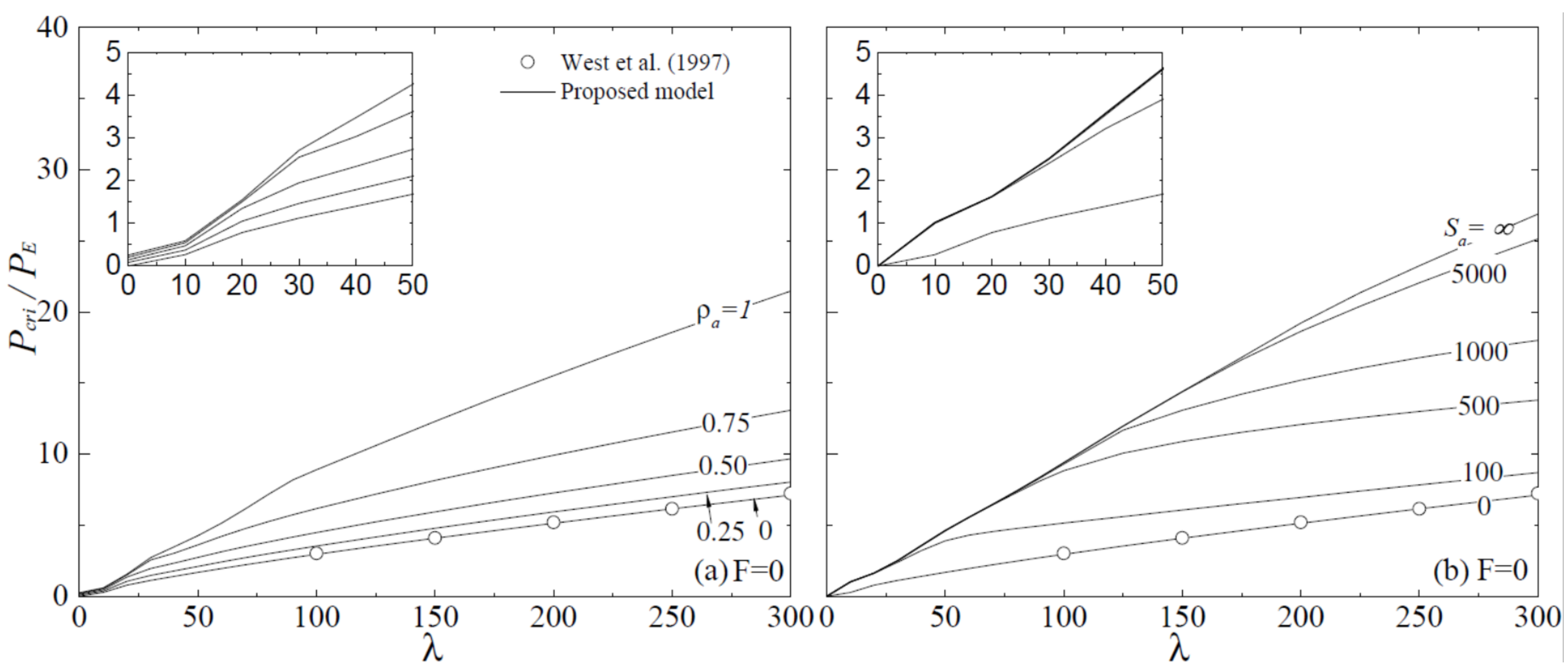
$$\bar{Y}(k+4) = \frac{1}{(k+4)(k+3)(k+2)(k+1)}\Big[\frac{L^3}{EI}(a_0\delta(k) + a_1\delta(k-1)L + a_2\delta(k-2)L^2) - (k+2)(k+1)F\bar{Y}(k+2) - \sum_{r=0}^{k}\bar{\Lambda}(k-r)\bar{Y}(r)\Big]$$

Parametric Analysis

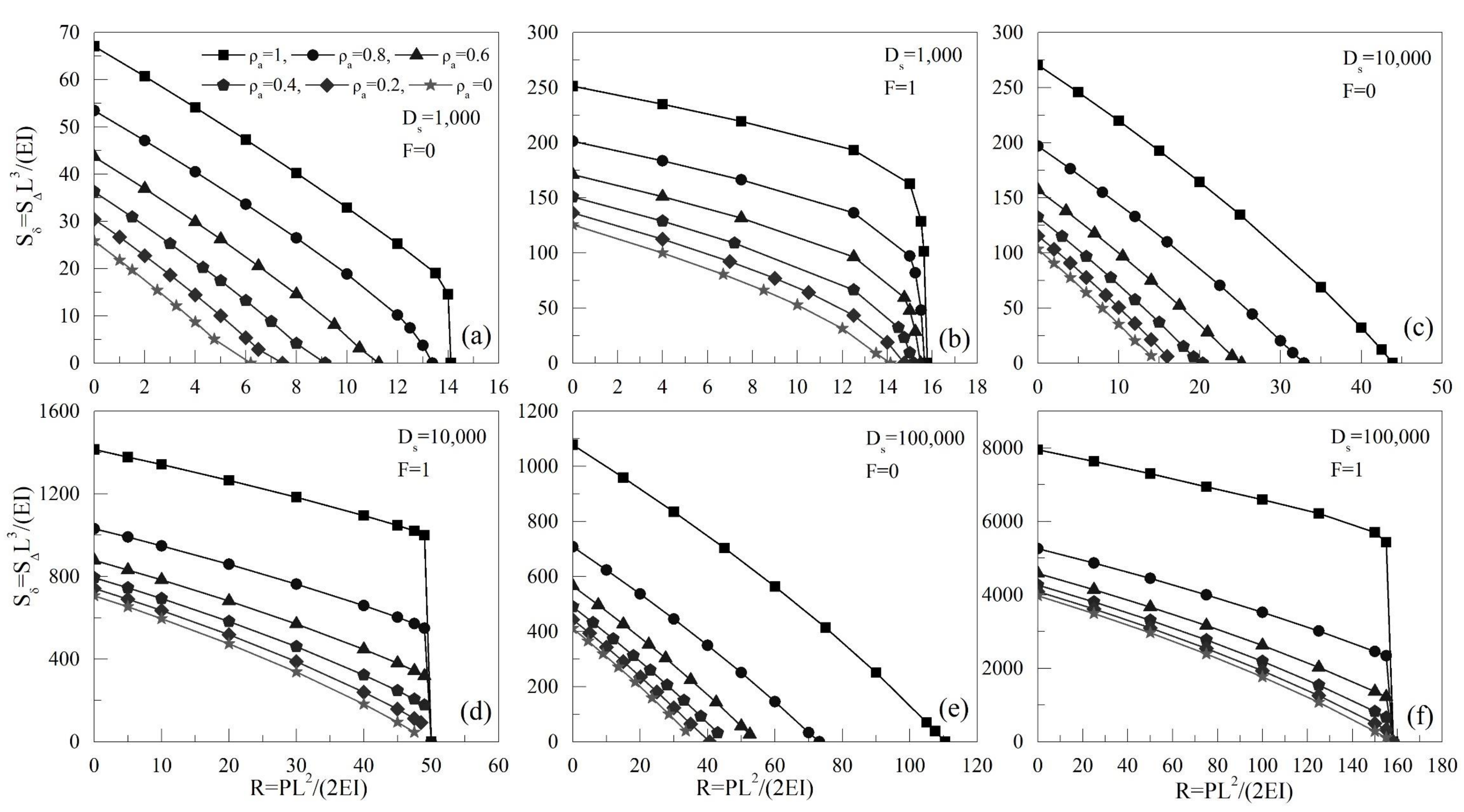


Variation in critical load for a free-free pile. F=0 (triangular linear distribution of k_s and F=1 (uniform distribution of k_s)

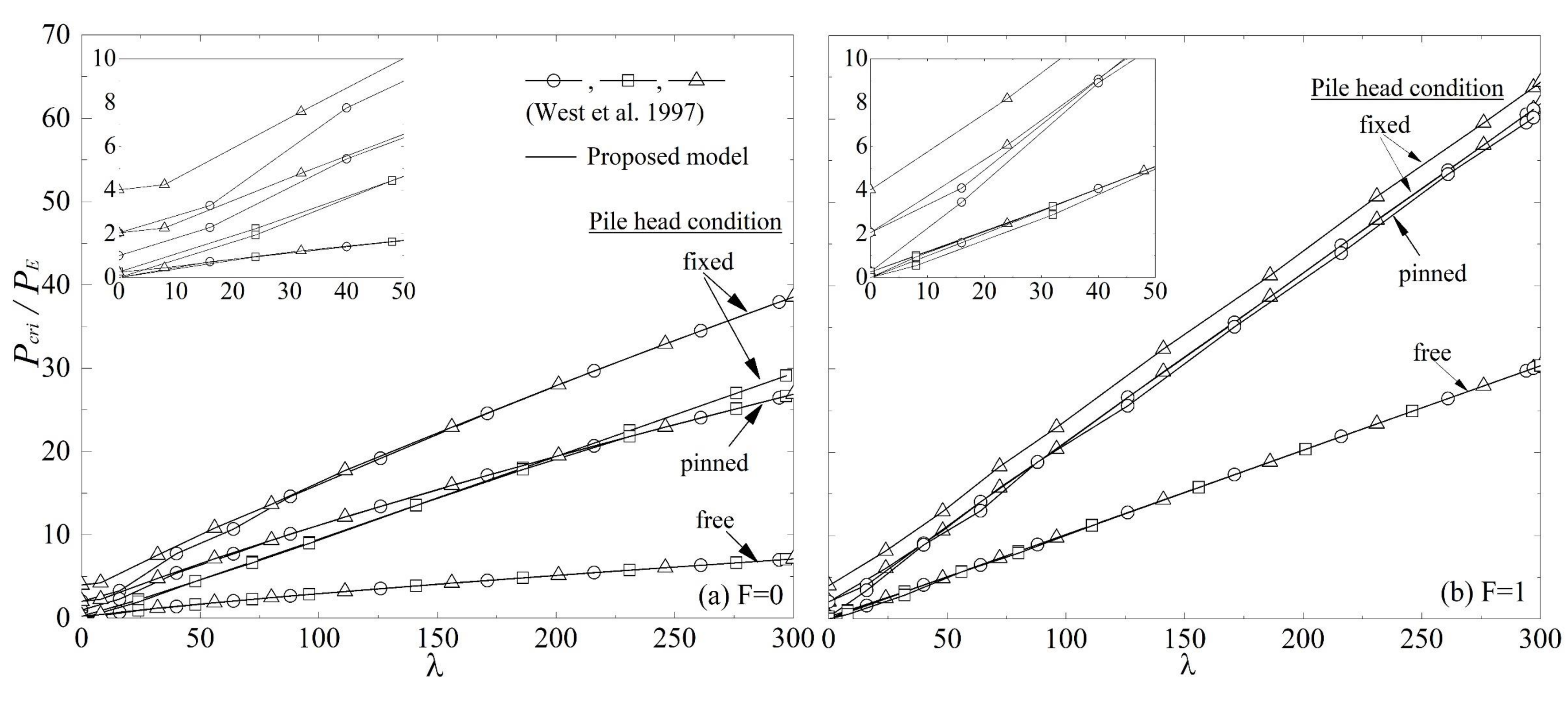
$$F = k_o/k_l$$
$$\lambda = (k_lL^4/EI)^{0.5}$$
$$S_\delta = \frac{S_\Delta L^3}{EI}$$
$$R = \frac{PL^2}{2EI}$$
$$D_s = \frac{K_sL^4}{EI}$$



Effects of a (a) rotational and (b) translational restraint at the pile head on the critical load for a pile free at the bottom and embedded in a soil with F=0.



Variation of S_Δ with R when (a) F=0 and (b) F=1



Critical load for a pile in a soil with (a) F=0 and (b) F=1 and various end-boundary conditions. (△) fixed; (◯) hinged; and (□) free.

Conclusions: A new, analytical approach to investigate the effect of generalized end-boundary conditions (ρ and *S*) and soil non-homogeneity (F) on the pile's critical load (P_{crit}) and second-order lateral stiffness (S_Δ) was developed. The well-known DTM Method was implemented to find the solution to the GDE in a compact and easy manner. Finding the solution to this GDE using conventional approaches is a very complex and cumbersome task. The results from the proposed model were validated against results from already available, but more limited in scope, analytical approaches. The agreement was excellent.