Study of forced vibrations of a two-layer plate under harmonic load

Vitaly Paimushin[1,2], Vyacheslav Firsov[1], Victor Shishkin[3], and Ruslan Gazizullin[1]

[1]*Kazan National Research Technical University named after A.N. Tupolev, Kazan, Russia;*
[2]*Kazan (Volga region) Federal University, Kazan, Russia*; [3]*Vyatka State University, Kirov, Russia.*

1. Rectangular finite element with viscoelastic damping

A special rectangular finite element consisting of a rigid isotropic layer 1 and a low-rigid damping layer 2 was developed to simulate the dynamic response of a plate with a viscoelastic damping coating (Fig. 1).

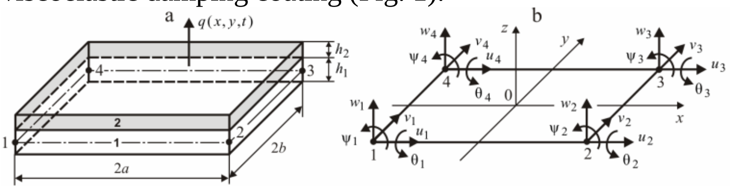

Fig. 1. Rectangular finite element of a two-layer plate

1.1 Approximation of displacements

Displacements u, v of an arbitrary point in the 0xy plane:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \mathbf{H}\mathbf{r}_\alpha, \quad \mathbf{H} = \begin{bmatrix} H_1 & 0 & H_2 & 0 & H_3 & 0 & H_4 & 0 \\ 0 & H_1 & 0 & H_2 & 0 & H_3 & 0 & H_4 \end{bmatrix},$$

$$\mathbf{r}_\alpha = \{u_1\ v_1\ u_2\ v_2\ u_3\ v_3\ u_4\ v_4\}.$$

Deflection w of the plate median surface:

$$w = \mathrm{N}^T \mathbf{r}_\beta, \quad \mathrm{N} = \{N_1\ \ N_2\ \ N_3\ \ \ldots\ \ N_{12}\},$$

$$\mathbf{r}_\beta = \{w_1\ \psi_1\ \theta_1\ w_2\ \psi_2\ \theta_2\ w_3\ \psi_3\ \theta_3\ w_4\ \psi_4\ \theta_4\}.$$

1.2 Geometric dependencies

$$\varepsilon_x = \frac{1}{a}\frac{\partial u}{\partial \xi} - z\frac{1}{a^2}\frac{\partial^2 w}{\partial \xi^2}, \quad \varepsilon_y = \frac{1}{b}\frac{\partial v}{\partial \eta} - z\frac{1}{b^2}\frac{\partial^2 w}{\partial \eta^2},$$

$$\gamma_{xy} = \frac{1}{a}\frac{\partial v}{\partial \xi} + \frac{1}{b}\frac{\partial u}{\partial \eta} - \frac{2z}{ab}\frac{\partial^2 w}{\partial \xi \partial \eta}, \quad \xi = x/a, \quad \eta = y/b.$$

1.3 Physical dependencies

The material of the rigid and damping layers of the plate is considered isotropic. To account for the elastic and damping properties of the material, the following physical dependences are used

$$\boldsymbol{\sigma}_k = \mathbf{D}_k \boldsymbol{\varepsilon} + \mathbf{D}_{g,k}\dot{\boldsymbol{\varepsilon}}$$

2. Formation and solution of the system of solving equations

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} = \mathbf{P}(t).$$

Here $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, $\mathbf{r}$, $\mathbf{P}(t)$ are the mass matrix, damping matrix, stiffness matrix, vector of nodal displacements, and vector of external nodal forces of the finite element model of the plate, respectively.

To solve the system we use the decomposition of the vector $\mathbf{r}$ by the eigenforms of oscillations: $\mathbf{r} = \mathbf{F}\,\mathbf{s}(t)$ where $\mathbf{F}$ is a rectangular matrix whose columns are m lower eigenforms, $\mathbf{s}(t)$ is the vector of generalized coordinates. This gives a system of linear algebraic equations (here $\mathbf{Q}_0 = \mathbf{F}^T\mathbf{P}_0$)

$$\begin{bmatrix} \mathbf{K}_\mathrm{F} - p^2\mathbf{M}_\mathrm{F} & p\mathbf{C}_\mathrm{F} \\ p\mathbf{C}_\mathrm{F} & -\mathbf{K}_\mathrm{F} + p^2\mathbf{M}_\mathrm{F} \end{bmatrix} \begin{Bmatrix} \mathbf{s}_a \\ \mathbf{s}_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{Q}_0 \\ 0 \end{Bmatrix}.$$

$s_{a,k} = s_{0,k}\cos\phi_k$, $s_{b,k} = s_{0,k}\sin\phi_k$ (ϕ_k is phase shift of vector $\mathbf{s}_0$ components with respect to vector $\mathbf{Q}_0$ components).

From here we find the vector elements $\mathbf{s}_0$ and the tangents of the angles ϕ_k:

$$s_{0,k} = \sqrt{s_{a,k}^2 + s_{b,k}^2}, \quad \mathrm{tg}\,\phi_k = s_{b,k}/s_{a,k}.$$

3. Numerical experiments

A rectangular simly supported plate with dimensions 480×560 mm, consisting of a rigid and a viscoelastic layer, under surface loading $q(t) = q_0 e^{ipt}$ with amplitude q_0 and frequency $p = 2\pi f$ at f varying from 0 to 200 Hz, is considered. Material of the hard layer is aluminum alloy D16AT, material of the viscoelastic layer is technical rubber.

Table 1. Natural frequencies of plate vibrations

j	f_j, Hz	j	f_j, Hz
1	11.2816	7	72.1762
2	25.9891	8	81.2843
3	31.2542	9	89.8884
4	46.1702	10	107.8782
5	51.6157	11	111.0019
6	66.0542	12	116.0356

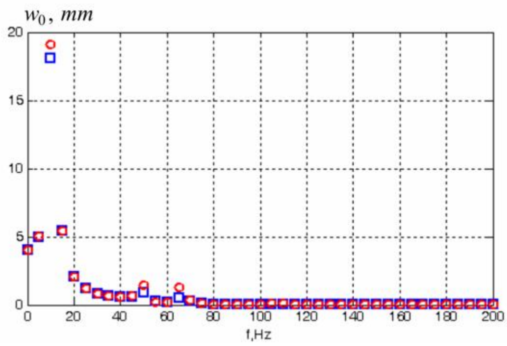

Fig. 2. Amplitudes of deflections w_0 of the plate center: squares are results with damping; circles are results without damping

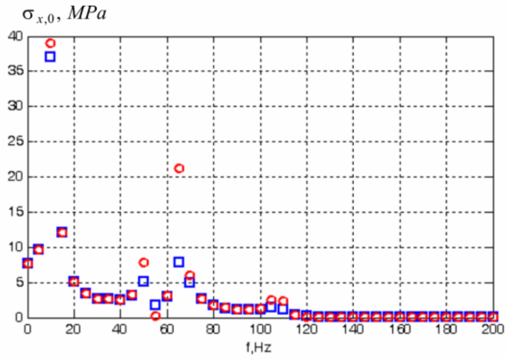

Fig. 2. Amplitudes of normal stresses $\sigma_{x,0}$ on the lower surface in the center of the plate: squares are results with damping; circles are results without damping

It can be seen from Figs. 2-3 that a noticeable difference between the results of calculating the plate with and without damping is observed only near the resonance frequencies, and far from resonance the results in both cases are almost the same. From the surface load $q(t) = q_0 e^{ipt}$, applied over the entire area of the plate, the resonance vibrations are excited only at those frequencies to which correspond eigenforms with an odd number of half-waves in the direction of each side of the plate. Of the twelve eigenfrequencies found, these frequencies are f_1, f_5, f_6 and f_{10}.

This research was supported by a grant from the Russian Science Foundation (project № 19-19-00058)