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Fracture Fatigue and Wear

Elastoplastic Bending Behaviors of Steel Plates via 2D Isogeometric Analysis Solution with Higher-Order Shear Deformation Theory

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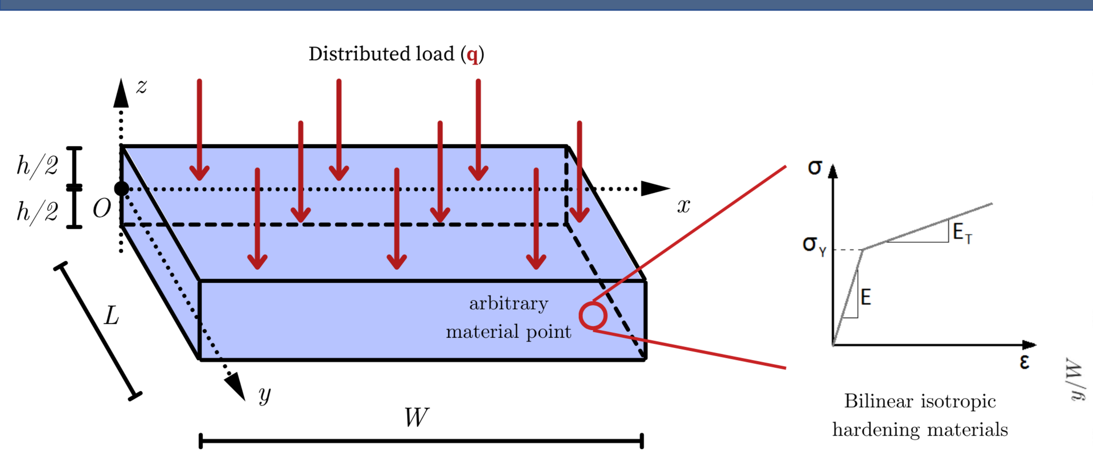
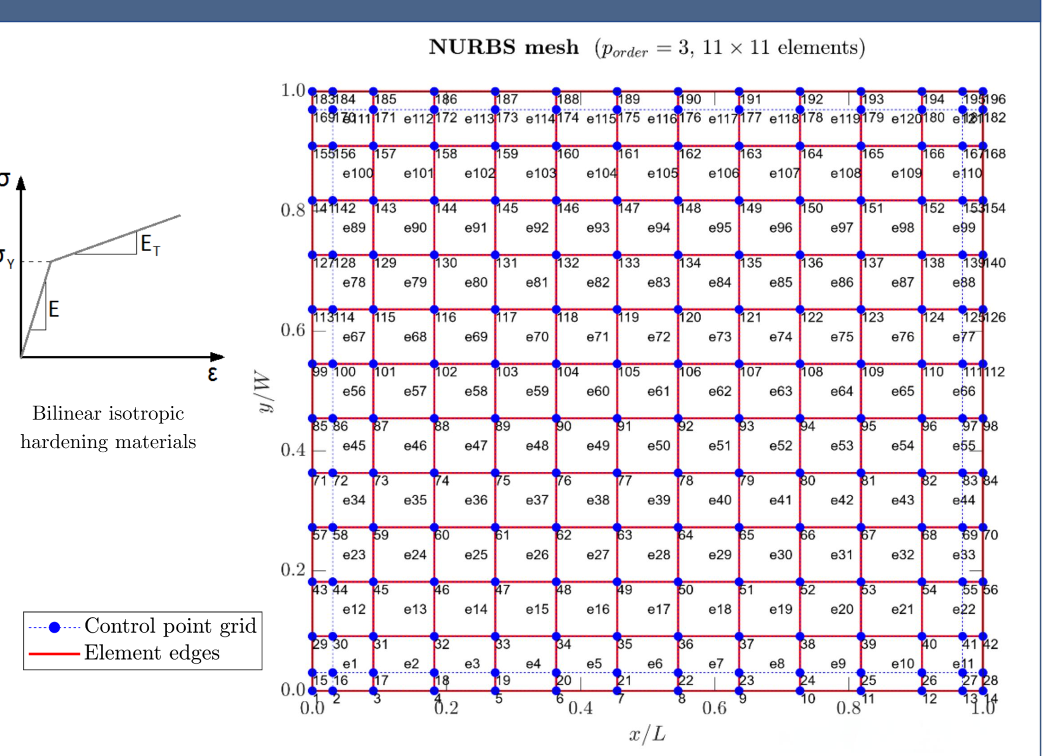
Introduction

Predicting the plasticity failure of metallic plates under transverse bending load is critical for structural integrity yet traditionally requires high computational costs.

To address this challenge, this work presents a computationally efficient, isogeometric analysis (IGA) approach combined with a four-variable higher-order shear deformation theory (HSDT) to analyze complex elastoplastic responses. By using exact CAD geometry and incorporating the von Mises yield condition with isotropic hardening rules, this high-continuity approach seamlessly captures plastic zone evolution and transverse shear dominance without the severe computational burden of full 3D continuum models.

Methods





HSDT Refined plate theory

$$\begin{cases} u(x,y,z) = u_0(x,y) - z\dfrac{\partial w_b(x,y)}{\partial x} + (f(z)-z)\dfrac{\partial w_s(x,y)}{\partial x}; \\ v(x,y,z) = v_0(x,y) - z\dfrac{\partial w_b(x,y)}{\partial y} + (f(z)-z)\dfrac{\partial w_s(x,y)}{\partial y}; \\ w(x,y,z) = w_b(x,y) + w_s(x,y). \end{cases}$$

Displacement field

$$\boldsymbol{u} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \{1 \quad z \quad (f(z)-z)\} \begin{Bmatrix} \boldsymbol{u_0} \\ \boldsymbol{u_1} \\ \boldsymbol{u_2} \end{Bmatrix}$$

where $\boldsymbol{u_0} = \{u_0 \quad v_0 \quad (w_b + w_s)\}^T$

$$\boldsymbol{u_1} = \left\{-\frac{\partial w_b(x,y)}{\partial x} \quad -\frac{\partial w_s(x,y)}{\partial y} \quad 0\right\}^T$$

$$\boldsymbol{u_2} = \left\{-\frac{\partial w_s(x,y)}{\partial x} \quad -\frac{\partial w_s(x,y)}{\partial y} \quad 0\right\}^T$$

Strain field

$$\boldsymbol{\varepsilon_b} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \tau_{xy} \end{Bmatrix} = \{1 \quad z \quad (f(z)-z)\} \begin{Bmatrix} \boldsymbol{\varepsilon_0} \\ \boldsymbol{\varepsilon_1} \\ \boldsymbol{\varepsilon_2} \end{Bmatrix}; \quad \boldsymbol{\gamma} = \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \frac{df(z)}{dz}\{\boldsymbol{\varepsilon_s}\}$$

where

$$\boldsymbol{\varepsilon_0} = \left\{\frac{\partial u_0}{\partial x} \quad \frac{\partial v_0}{\partial y} \quad \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right)\right\}^T; \quad \boldsymbol{\varepsilon_1} = \left\{-\frac{\partial^2 w_b}{\partial x^2} \quad -\frac{\partial^2 w_b}{\partial y^2} \quad -2\frac{\partial^2 w_b}{\partial x \partial y}\right\}^T;$$

$$\boldsymbol{\varepsilon_2} = \left\{\frac{\partial^2 w_s}{\partial x^2} \quad \frac{\partial^2 w_s}{\partial y^2} \quad 2\frac{\partial^2 w_s}{\partial x \partial y}\right\}^T; \boldsymbol{\varepsilon_s} = \left\{\frac{\partial w_s}{\partial y} \quad \frac{\partial w_s}{\partial x}\right\}^T.$$

NURBS approximation of Displacement and Strain fields

$$\tilde{\boldsymbol{u}}(\xi,\eta) = \sum_{e=1}^{n_\xi \times n_\eta} \mathbf{I}_{4x4}\, \mathbf{R}_e(\xi,\eta)\mathbf{d}_e$$

where $\tilde{\boldsymbol{u}}(\xi,\eta) = \{u_0 \quad v_0 \quad w_b \quad w_s\}^T$

$$\mathbf{d}_e = \{u_{0,e} \quad v_{0,e} \quad w_{b,e} \quad w_{s,e}\}^T$$

$$R_I(\xi,\eta) = \frac{N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{\sum_{i=1}^{n_\xi}\sum_{j=1}^{n_\eta} N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}},$$

$$\tilde{\boldsymbol{\varepsilon}}(\xi,\eta) = \begin{Bmatrix} \boldsymbol{\varepsilon_0}(\xi,\eta) \\ \boldsymbol{\varepsilon_1}(\xi,\eta) \\ \boldsymbol{\varepsilon_2}(\xi,\eta) \\ \boldsymbol{\varepsilon_s}(\xi,\eta) \end{Bmatrix} = \sum_{e=1}^{n_\xi \times n_\eta} \begin{Bmatrix} \boldsymbol{B_0}(\xi,\eta) \\ \boldsymbol{B_1}(\xi,\eta) \\ \boldsymbol{B_2}(\xi,\eta) \\ \boldsymbol{B_s}(\xi,\eta) \end{Bmatrix} \mathbf{d}_e$$

$$\boldsymbol{B_0} = \begin{bmatrix} \frac{\partial R_e}{\partial x} & 0 & 0 & 0 \\ 0 & \frac{\partial R_e}{\partial y} & 0 & 0 \\ \frac{\partial R_e}{\partial y} & \frac{\partial R_e}{\partial x} & 0 & 0 \end{bmatrix}; \boldsymbol{B_1} = \begin{bmatrix} 0 & 0 & -\frac{\partial^2 R_e}{\partial x^2} & 0 \\ 0 & 0 & -\frac{\partial^2 R_e}{\partial y^2} & 0 \\ 0 & 0 & -2\frac{\partial^2 R_e}{\partial x \partial y} & 0 \end{bmatrix}; \boldsymbol{B_2} = \begin{bmatrix} 0 & 0 & 0 & \frac{\partial^2 R_e}{\partial x^2} \\ 0 & 0 & 0 & \frac{\partial^2 R_e}{\partial y^2} \\ 0 & 0 & 0 & 2\frac{\partial^2 R_e}{\partial x \partial y} \end{bmatrix}; \boldsymbol{B_s} = \begin{bmatrix} 0 & 0 & 0 & \frac{\partial R_e}{\partial y} \\ 0 & 0 & 0 & \frac{\partial R_e}{\partial x} \end{bmatrix};$$

[1] Hughes et. Al., Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, Computer Methods in Applied Mechanics and Engineering 194 (2005) 4135–4195.

IGA solution for elasto-plastic HSDT bending plate analysis

$$\int_\Omega \int_{-h/2}^{h/2} \boldsymbol{\sigma}_{n+1}^T \delta\boldsymbol{\varepsilon}\, dzd\Omega - \int_\Omega \int_{-h/2}^{h/2} q_{n+1}\delta w\, dzd\Omega = 0 \qquad (\text{at state } t_{n+1})$$

$$\Leftrightarrow \int_\Omega \int_{-h/2}^{h/2} (\boldsymbol{\sigma}_n^T + \Delta\boldsymbol{\sigma}^T)\delta\boldsymbol{\varepsilon}\, dzd\Omega - \int_\Omega \int_{-h/2}^{h/2} q_{n+1}\delta w\, dzd\Omega = 0$$

$$\Leftrightarrow \int_\Omega \int_{-h/2}^{h/2} \Delta\boldsymbol{\sigma}^T \delta\boldsymbol{\varepsilon}\, dzd\Omega = \int_\Omega \int_{-h/2}^{h/2} q_{n+1}\delta w\, dzd\Omega - \int_\Omega \int_{-h/2}^{h/2} \boldsymbol{\sigma}_n^T \delta\boldsymbol{\varepsilon}\, dzd\Omega$$

$$\Leftrightarrow \int_\Omega \int_{-h/2}^{h/2} \Delta\boldsymbol{\varepsilon}^T \mathbf{C}^{ep} \delta\boldsymbol{\varepsilon}\, dzd\Omega = \int_\Omega \int_{-h/2}^{h/2} q_{n+1}\delta w\, dzd\Omega - \int_\Omega \int_{-h/2}^{h/2} \boldsymbol{\sigma}_n^T \delta\boldsymbol{\varepsilon}\, dzd\Omega$$

$$\Leftrightarrow \int_\Omega \Delta\tilde{\boldsymbol{\varepsilon}}^T \left(\int_{-h/2}^{h/2} \widetilde{\mathbf{B}^T}\mathbf{Q}^{ep}\tilde{\mathbf{B}}\, dz\right)\delta\tilde{\boldsymbol{\varepsilon}}d\Omega = \int_\Omega q_{n+1}\delta w d\Omega - \int_\Omega \left(\left(\int_{-h/2}^{h/2} \boldsymbol{\sigma}_n^T \tilde{\mathbf{B}}\, dz\right)\delta\tilde{\boldsymbol{\varepsilon}}d\Omega\right.$$

$$\underbrace{\Leftrightarrow \int_\Omega \Delta\tilde{\boldsymbol{\varepsilon}}\tilde{\mathbf{B}}^T \left(\underbrace{\int_{-h/2}^{h/2} \begin{Bmatrix} 1 \\ z \\ f(z)-z \\ \frac{df(z)}{dz} \end{Bmatrix} \left\{1 \quad z \quad (f(z)-z) \quad \frac{df(z)}{dz}\right\} \mathbf{Q}^{ep}\, dz}\right) \tilde{\mathbf{B}}\delta\tilde{\boldsymbol{\varepsilon}}d\Omega = \int_\Omega q_{n+1}\delta w d\Omega - \int_\Omega \widetilde{\mathbf{B}^T} \left(\underbrace{\int_{-h/2}^{h/2} \begin{Bmatrix} 1 \\ z \\ f(z)-z \\ \frac{df(z)}{dz} \end{Bmatrix} \boldsymbol{\sigma}_n\, dz}\right)\delta\tilde{\boldsymbol{\varepsilon}}d\Omega}$$

Integration along the plate thickness (z-direction) at Oxy Gaussian quadrature points

Integration over the plate domain (Oxy plane), forming $\mathbf{K}^{ep}\Delta\mathbf{U} = \mathbf{F}_{ext}^{n+1} - \mathbf{F}_{int}^{n}$

Elasto-plastic Stress-Strain model

- Elasto-plastic strain: $d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^e + d\boldsymbol{\varepsilon}^p$
- Yield surface: $f(\boldsymbol{\sigma}, \bar{\varepsilon}_{eq}^p) = \sigma_{vm} - (\sigma_y + H\bar{\varepsilon}_{eq}^p)$
- Associated flow rule:

$$d\boldsymbol{\varepsilon}^p = d\gamma\frac{\delta f}{\delta\boldsymbol{\sigma}} \text{ with } d\gamma = \frac{f}{3G+H}$$

- Elasto-plastic tangent modulus ($\mathbf{C}^{ep}$): $d\boldsymbol{\sigma} = \mathbf{C}^{ep}d\boldsymbol{\varepsilon}$
- Radial return mapping: $\boldsymbol{\sigma}_{tr} = \boldsymbol{\sigma}_n + \mathbf{C}^e\Delta\boldsymbol{\varepsilon}$

$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_{tr} - \left(\frac{3G\Delta\gamma}{\sigma_{vm,tr}}\right)\boldsymbol{s}_{tr}$$

$$\mathbf{C}^{ep} = \mathbf{C}^e - \frac{9G^2}{\sigma_{vm,tr}^2(3G+H)}(\boldsymbol{s}_{tr}\boldsymbol{s}_{tr}^T)$$

$$\bar{\varepsilon}_{eq,n+1}^p = \bar{\varepsilon}_{eq,n}^p + \Delta\gamma$$

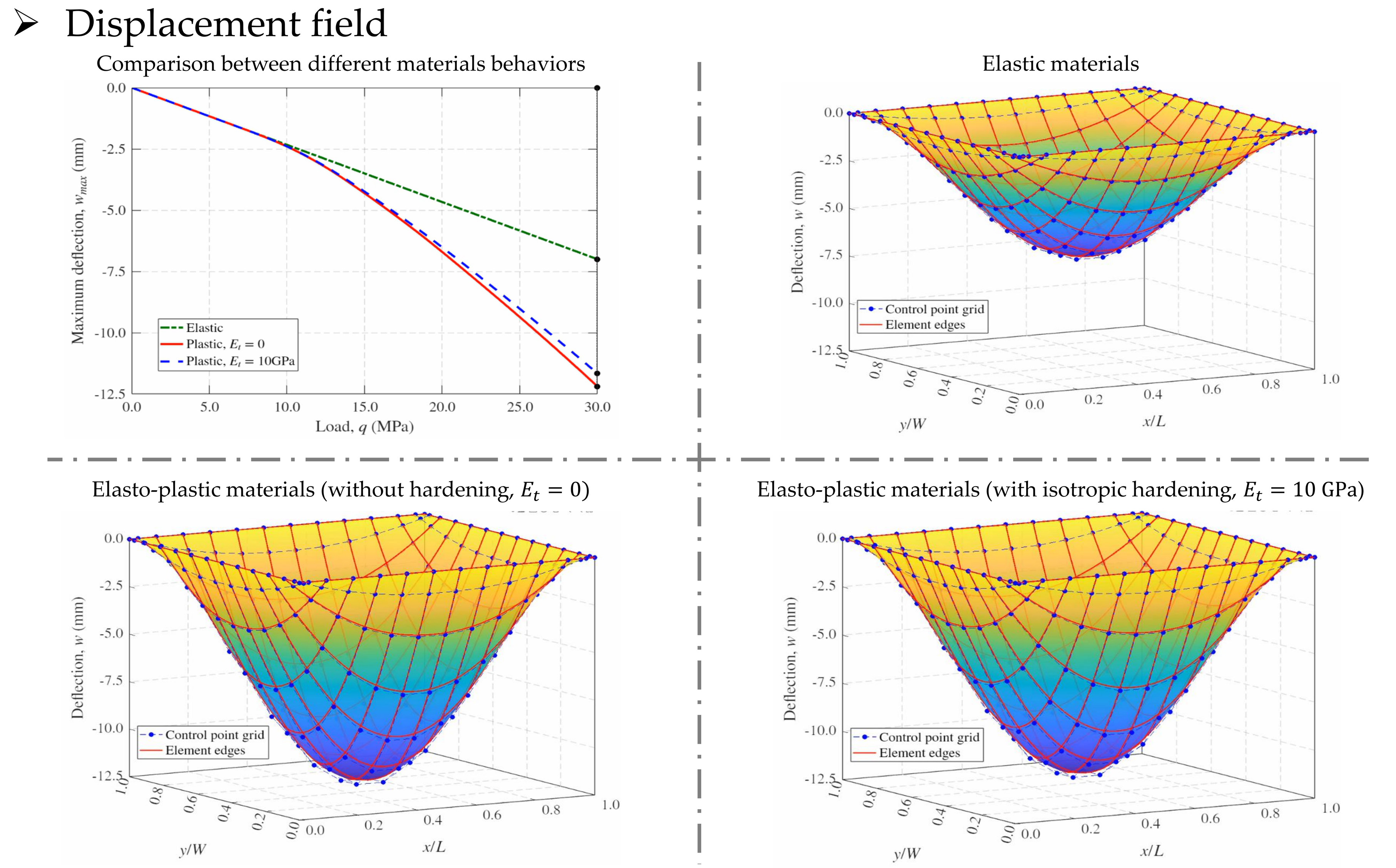
HSDT model correction of the elasto-plastic tangent modulus

$$d\sigma_i = C_{ij}^{ep}d\varepsilon_j \xrightarrow[d\varepsilon_3 = 0]{d\sigma_3 = 0} d\sigma_3 = \sum_{j=1}^{6} C_{3j}^{ep}d\varepsilon_j = 0 \Rightarrow d\varepsilon_3 = -\frac{1}{C_{33}^{ep}}\sum_{\substack{j=1 \\ j\neq 3}}^{6} C_{3j}d\varepsilon_j \longrightarrow d\sigma_i = \sum_{\substack{j=1 \\ j\neq 3}}^{6}\left(C_{ij}^{ep} - \frac{C_{i3}^{ep}C_{3j}^{ep}}{C_{33}^{ep}}\right)d\varepsilon_j \longrightarrow \mathbf{Q}^{ep} = \mathbf{C}^{ep} - \frac{\mathbf{C}_{i3}^{ep}\mathbf{C}_{3j}^{ep}}{\mathbf{C}_{33}^{ep}}$$

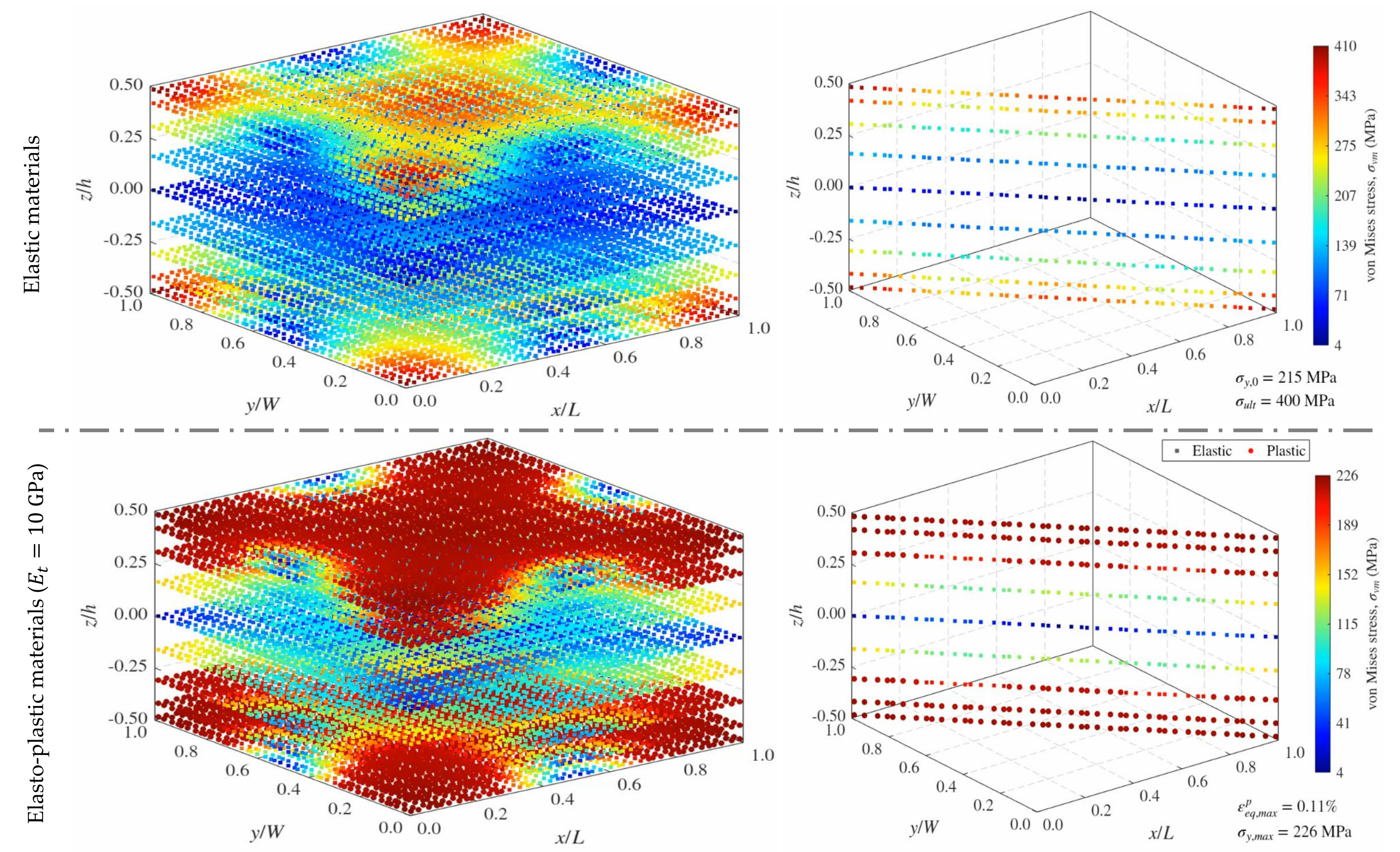
Numerical examples

- Geometry: **Thick Square** ($L = W = 1$ m, $h = a/10 = 0.1$ m)
- Shear deformation function: **Reddy** ($f_z = z - 4z^3/(3h^2)$)
- Material: **SUS304** ($E = 200$ GPa, $\nu = 0.3$, $\sigma_y = 215\ MPa$, $E_t = 10\ GPa$)
- Boundary conditions: **Full simply supported** (at $x = 0$ or L, $v_0 = w_b = w_s = 0$, at $y = 0$ or W, $u_0 = w_b = w_s = 0$)
- Load: **Incremental Uniformly distributed** ($q = 30$ MPa)

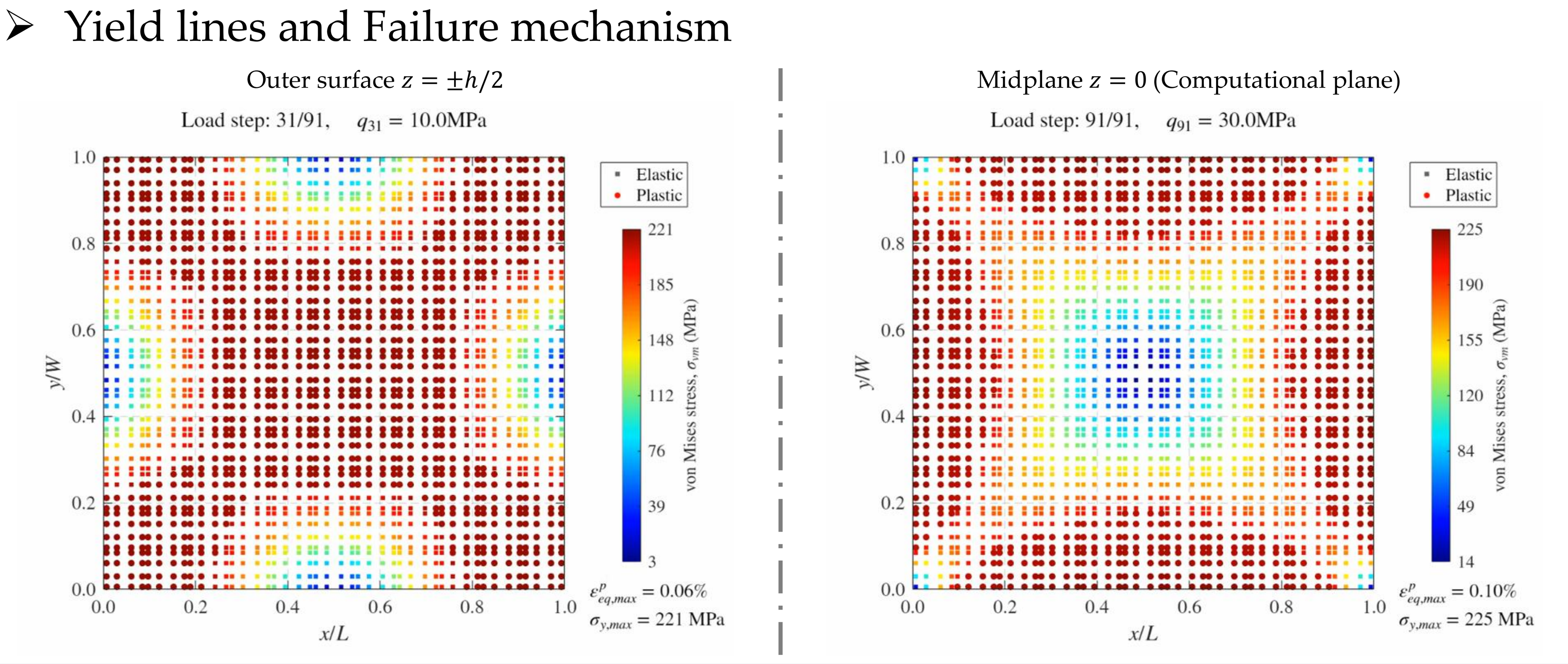
➢ Displacement field



➢ von Mises Stress field ($q = 12.5$ MPa)



➢ Yield lines and Failure mechanism



Conclusions

➢ The HSDT-IGA framework efficiently captures critical out-of-plane effects without the high computational cost of full 3D continuum models.

➢ Higher-order spline basis functions ensure exact CAD fidelity and smooth continuity, stabilizing convergence during severe elastoplastic deformation.

➢ Outer-surface yielding is driven by bending, while midplane yielding is dominated by transverse shear, a critical transition underestimated by plane stress theories.