R-Calculus For Post Three-Valued Description Logic

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Introduction

In many-valued logic, it is important to give an explanation of the truth-values other than the truth **t** and the falsity **f**. For example, in a three-valued logic , the third value **m** is interpreted as unknown or indeterminate, and the definition of binary logical connectives are independent of **m**.

Description logics are different, because a concept seems natural to have different counterparts.

R-calculi is a belief revision operator satisfying AGM postulates, and a deduction system for enumerating a formula A into a consistent theory Δ to keep the theory $\mathbf{A}',\Delta$ consistent as possible. A condition that there is a sound and complete **R**-calculus is that the based logic is decidable.

Description logics are fragments of first-order logic, some of which are decidable and some are not. We consider one of many-valued description logics: Post three-valued
description logics, where the logical language of Post logic contains a unary connective ~, instead of ¬. Because for these logics there are sound and complete tableau proof systems, Gentzen deduction systems and deduction systems for many-placed sequents. For decidable description logics, a problem is to define the semantics of quantifier concept constructors. In binary ones, an element a belongs to interpretation of concept $(\forall R.C)$ if for any element b with $(a,b)\in R^I$, $b\in C^I$; and an element a belongs to interpretation of concept $\neg(\forall R.C)$ if for some element b with $(a,b)\in R^I$, $b\notin C^I$. Correspondingly, we define the element in Post three-valued description logic.

A theory (a set of statements) Δ is t-satisfiable if there is an interpretation $\boldsymbol{I}$ such that for any statement $C(a)\in\Delta$, $(C(a))^I\neq\mathbf{t}$.

We will give a tableau proof system $\mathbf{T}_t$ for t-satisfiability, which is sound, complete and nonmonotonic.

Based on the tableau proof system $\mathbf{T}_t$, we construct an **R**-calculus $\mathbf{R}_t$ for $\Delta|A\Rightarrow A',\Delta$. $\mathbf{R}_t$ is shown to be sound and complete.

Methods

- Post three-valued description logic
- Nonmonotonic tableau proof system
- R-calculus

Conclusions

This paper gave an **R**-calculus $\mathbf{R}_t$ for t-satisfiability in Post three-valued description logic, which is sound and complete.

Similarly there are **R**-calculi $\mathbf{R}_m$ and $\mathbf{R}_f$ for m-satisfiability and f-satisfiability, respectively, and there are transformations between$\mathbf{R}_t$, $\mathbf{R}_m$ and $\mathbf{R}_f$ just as transformations $\mathbf{T}_t$, $\mathbf{T}_m$ and $\mathbf{T}_f$.