A Limit Theorem for Weighted Sums of Random Sets in Fuzzy Metric Space

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Introduction

- In 1965, Zadeh introduce the fuzzy set theory.
- In 1994, George and Veeramani introduced the definition of fuzzy metric for single-valued variables and discussed the completeness and separability in fuzzy metric space.
- In 2005, Saadati and Vaezpour defined fuzzy normed space.
- In 2011, Gregori and Morillas introduced examples of fuzzy metrics and its applications.
- In 2021, Guan etc. introduced the definition of fuzzy metric for sets, discussed the properties and proved the laws of large numbers for random sets in the sense of fuzzy metric.

Preliminaries

Throughout this paper, we assume that $(\Omega, \mathcal{A}, \mu)$ is a complete probability space, $(\mathfrak{X}, \|\cdot\|)$ is a real separable Banach space, $\mathbf{K}(\mathfrak{X})(\mathbf{K}_k(\mathfrak{X}))$ is the family of all nonempty closed (resp.compact) subsets of $\mathfrak{X}$, and $\mathbf{K}_{kc}(\mathfrak{X})$ is the family of all nonempty compact convex subsets of $\mathfrak{X}$.

Definition Let * be a continuous t-norm. The 3-tuple $(K(\mathfrak{X}), M, *)$ is said to be a fuzzy metric space for sets, if the mapping $M: \mathrm{K}(\mathfrak{X}) \times \mathrm{K}(\mathfrak{X}) \times (0,\infty)$ satisfies the following conditions, $\forall A, B, C \in \mathrm{K}(\mathfrak{X})$ and t, s>0:

(1) $\forall t > 0, M(A,B,t) > 0;$
(2) $\forall t > 0, M(A,B,t) = 1 \Leftrightarrow A = B;$
(3) M(A,B,t)=M(B,A,t);
(4) $M(A,B,t) * M(B,C,s) \leq M(A,C,t+s);$
(5) $M(A,B,\cdot) : (0,\infty) \to [0,1]$ is continuous.

M is called a **fuzzy metric** on $\mathrm{K}(\mathfrak{X})$.

Methods

Lemma 1 Let $\mathfrak{X}$ be a separable normed space. There exists a fuzzy normed space $C(S^*)$ and a function $j_0: K_{kc}(\mathfrak{X}) \to C(S^*)$ with the following properties: for $A, B \in K_{kc}(\mathfrak{X})$, t>0

$$(1) M_{d_H}(A,B,t) = M_d(j_0(A), j_0(B), t);$$
$$(2) j_0(A+B) = j_0(A) + j_0(B);$$
$$(3) j_0(\lambda A) = \lambda j_0(A), \lambda \geq 0,$$

Where M_d is the fuzzy metric induced by metric d in embedding space. Thus, $K_{kc}(\mathfrak{X})$ is embedded into a normed space by $j_0(\cdot)$.

Lemma 2 Let $\{V_n: n \geq 1\} \subset L^1[\Omega; K_k(\mathfrak{X})]$. The fuzzy metric M_{d_H} is induced by d_H, then for any t>0

$$M_{d_H}\left(\sum_{i=1}^{n} V_i, \sum_{i=1}^{n} coV_i, t\right) \geq \min_{1\leq i\leq n} M_{d_H}\left(coV_i, \{0\}, \frac{t}{\sqrt{p}}\right)$$

Conclusions

Theorem 1 Let $\{V_n: n \geq 1\} \subset L^1[\Omega; K_k(\mathfrak{X})]$ be an independent and compactly uniformly random sets. Let $\{b_n: n \geq 1\}$ be a sequence of positive constants with $b_n \uparrow$ and $n = O(b_n)$. If

$$\sum_{n=1}^{\infty} \frac{1}{b_n^p} E[\|V_n\|_K^p] < \infty, 1 \leq p \leq 2.$$

Then in the metric M_{d_H}, for t>0 we have the following convergence :

$$M_{d_H}\left(\frac{1}{b_n}\sum_{i=1}^{n} V_i, \frac{1}{b_n}\sum_{i=1}^{n} E[coV_i], t\right) \longrightarrow 1 \quad a.e..$$

In this paper, we gave the strong limit theory for weighted sums of compact random sets in the sense of fuzzy metric, where the fuzzy metric is induced by d_H.