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An Energy-based Physics-Informed Neural Networks Approach for Nonlinear Analysis of Bio-inspired TPMS Plates via Higher-order Shear Deformation Theory

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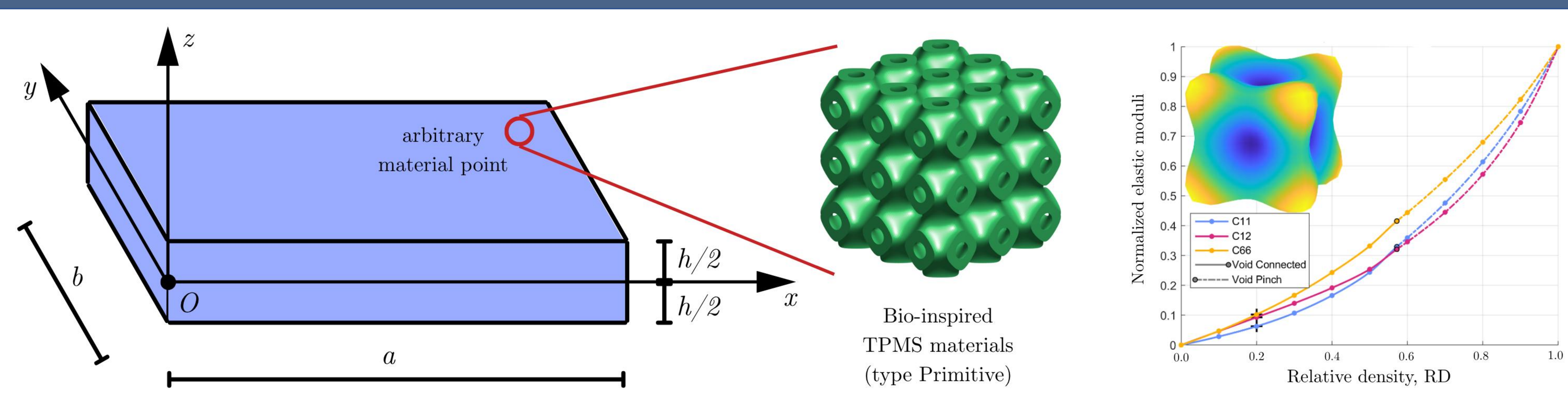
Introduction

Bio-inspired triply periodic minimal surface (TPMS) materials offer exceptional energy absorption and structural efficiency for aerospace applications, but modeling their geometrically nonlinear bending response remains a significant challenge.

To address this, this work introduces a computationally efficient, energy-based physics-informed neural network (ePINN) combined with a five-variable higher-order shear deformation theory (HSDT). By directly minimizing total potential energy without mesh-based discretization or iterative solvers, this framework provides a physically consistent tool for predicting the complex behavior of next-generation aerospace components.

Methods



Let: • $\mathbf{x} = \{x,\ y\}^T \in \mathbb{R}^2$ is the input coordinate vector.

• $\mathbf{y} = \{u_0,\ v_0,\ w_0,\ \beta_y,\ \beta_x\}^T \in \mathbb{R}^5$ is the output variable field vector.

$$\mathbf{y} = \mathbf{f}_{\mathbf{trial}}(\mathbf{x}) = \mathbf{g}_{\mathbf{p}}(\mathbf{x}) + \mathbf{f}_{\mathbf{dist}}(\mathbf{x}) \odot \mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x})$$

The trial function

A particular function enforcing the non-homogeneous boundary values

A distance function designed to vanish at the boundaries

Hadamard (element-wise) product

Raw output of the neural network:

$$\mathbf{f}_{\boldsymbol{\theta}}(\mathbf{x}) = (\mathbf{f}_L \circ \mathbf{f}_{L-1} \circ \cdots \circ \mathbf{f}_1)(\mathbf{x})$$

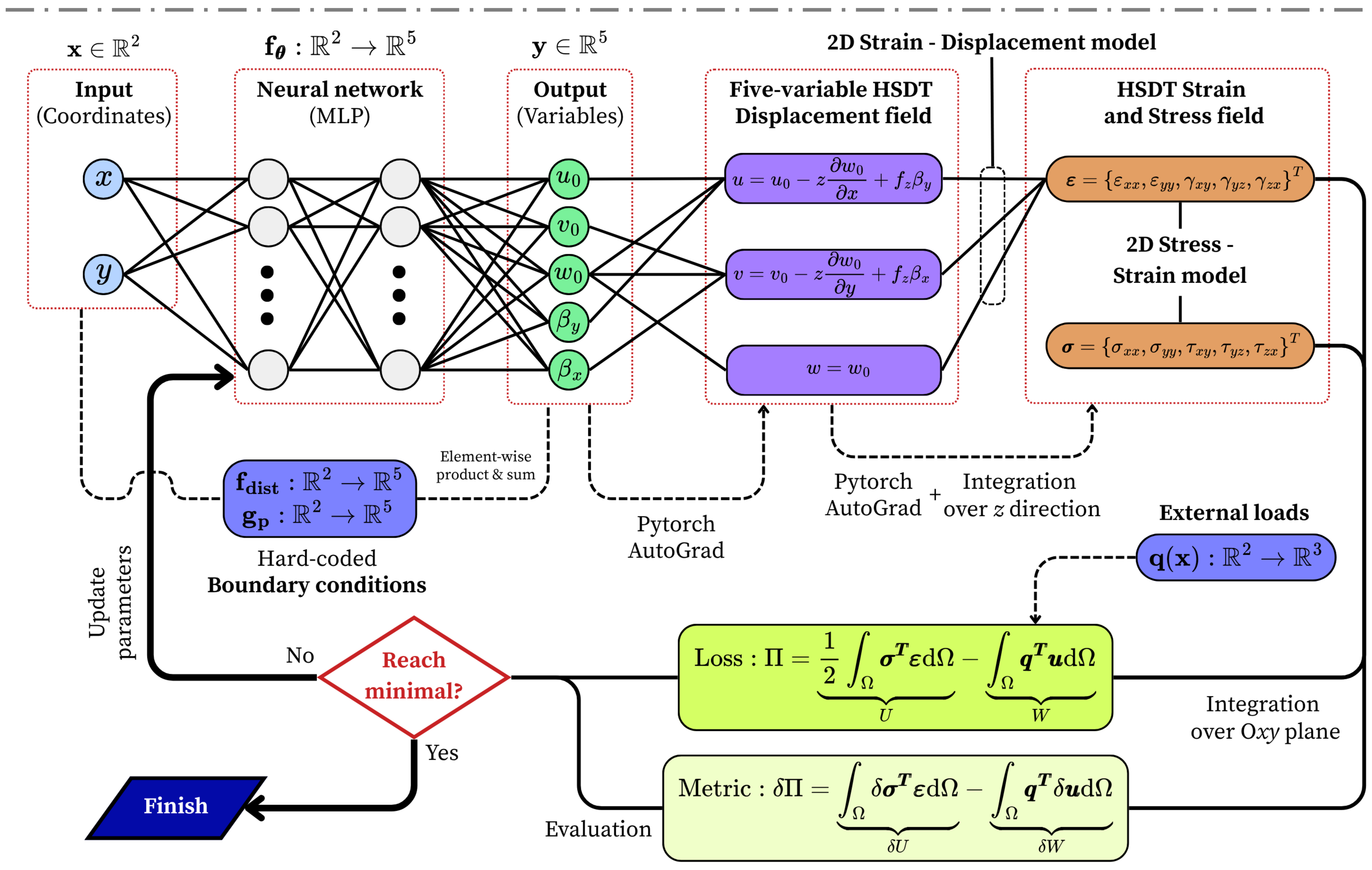
where: • $\mathbf{f}_l(\mathbf{x}) = \boldsymbol{\Phi}_l(\mathbf{W}_l \cdot \mathbf{x} + \mathbf{b}_l)$ is the l^{th} layer function.

• $\circ$ represents the function composition operator.

• $\boldsymbol{\Phi}_l(\cdot)$ is the fixed element-wise activation function at layer l.

• $\mathbf{W}_l \in \mathbb{R}^{d_l \times d_{l-1}}$ and $\mathbf{b}_l \in \mathbb{R}^{d_l}$ are weight matrix and bias vector of l^{th} layer, respectively.

• $\boldsymbol{\theta} = \{\mathbf{W}_l, \mathbf{b}_l\}_{l=1}^{L}$ are learnable parameters of the network.



2D Strain-Displacement model
(Green-Lagrange strain)

$$\begin{cases}
\varepsilon_{xx}(x,y,z) = \dfrac{\partial u_0}{\partial x} - z\dfrac{\partial^2 w_0}{\partial x^2} + f(z)\dfrac{\partial \beta_y}{\partial x} + \boxed{\dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2};\\
\varepsilon_{yy}(x,y,z) = \dfrac{\partial v_0}{\partial y} - z\dfrac{\partial^2 w_0}{\partial y^2} + f(z)\dfrac{\partial \beta_x}{\partial y} + \boxed{\dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2};\\
\gamma_{xy}(x,y,z) = \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} - 2z\dfrac{\partial^2 w_0}{\partial x \partial y} + f(z)\left(\dfrac{\partial \beta_y}{\partial y} + \dfrac{\partial \beta_x}{\partial x}\right) + \boxed{\dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y}};\\
\gamma_{yz}(x,y,z) = \dfrac{df(z)}{dz}\dfrac{\partial \beta_x}{\partial y};\\
\gamma_{xz}(x,y,z) = \dfrac{df(z)}{dz}\dfrac{\partial \beta_y}{\partial x};\\
\varepsilon_{zz}(x,y,z) = 0.
\end{cases}$$

Nonlinear components (large deformation)

2D Stress-Strain model
(Elastic cubic-symmetric constitutive)

$$\boldsymbol{\sigma} = \mathbf{C} \cdot \boldsymbol{\varepsilon}$$

$$\mathbf{C} = \begin{bmatrix}
\frac{E(1-\nu)}{(1+\nu)(1-2\nu)} & \frac{E\nu}{(1+\nu)(1-2\nu)} & \frac{E\nu}{(1+\nu)(1-2\nu)} & 0 & 0 & 0\\
\frac{E\nu}{(1+\nu)(1-2\nu)} & \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} & \frac{E\nu}{(1+\nu)(1-2\nu)} & 0 & 0 & 0\\
\frac{E\nu}{(1+\nu)(1-2\nu)} & \frac{E\nu}{(1+\nu)(1-2\nu)} & \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} & 0 & 0 & 0\\
0 & 0 & 0 & G & 0 & 0\\
0 & 0 & 0 & 0 & G & 0\\
0 & 0 & 0 & 0 & 0 & G
\end{bmatrix}$$

where $E(RD)$, $\nu(RD)$, and $G(RD)$ are provided in [1]

❖ At $RD = 0.4$, TPMS type P has $E/E_s = 0.153$, $\nu = 0.329$, and $G/G_s = 0.245 \Rightarrow$ Zenner's anisotropic index, $A_z = 1.640$

[1] Nguyen-Xuan et. Al., Modelling of functionally graded triply periodic minimal surface (FG-TPMS) plates, Composite Structures 315 (2023) 116981.

Numerical examples

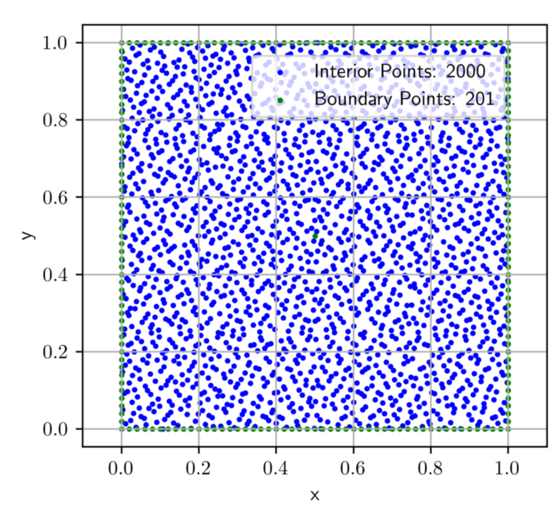
- Geometry: **Rectangular ($a = b = 1$ m, $h = a/10 = 0.1$ m)**
- Shear deformation function: **Reddy ($f_z = z - 4z^3/(3h^2)$)**
- Base material: **Metal ($E = 200$ GPa, $\nu = 0.3$)**
- Porous material: **Type Primitive (P), $RD = 0.40$**
- Boundary conditions: **Full simply supported**

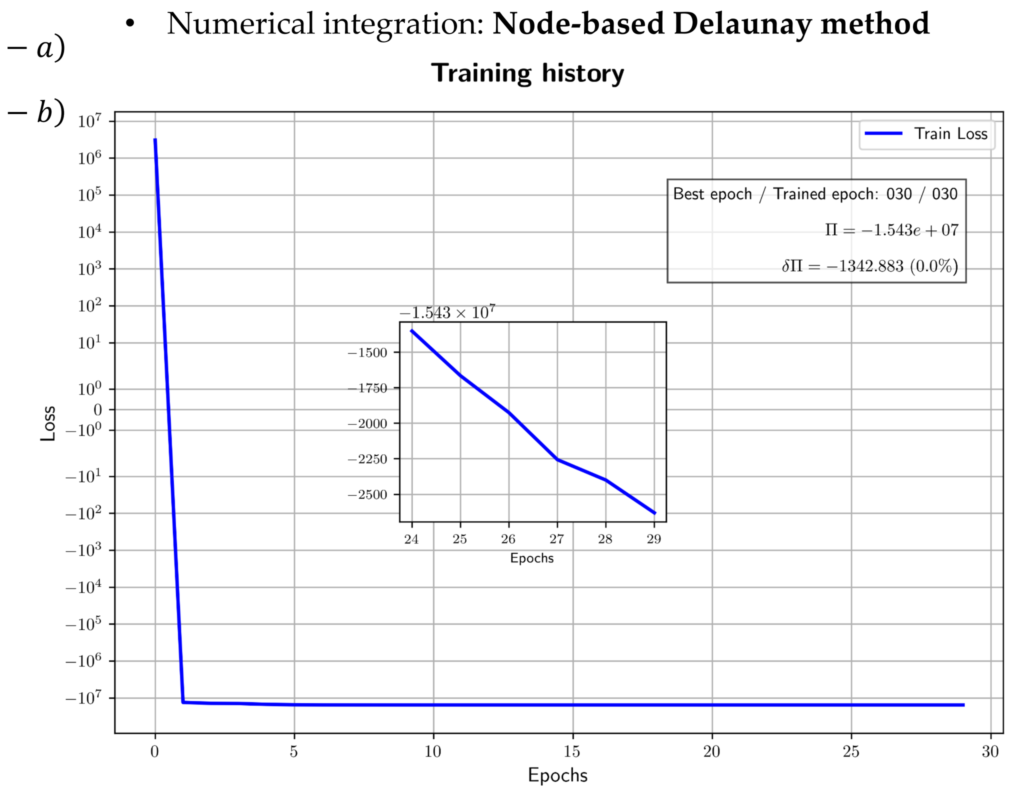
  at $x = 0$ or a, $\quad v_0 = \beta_x = w_0 = 0 \;\Rightarrow f_{dist}^{v_0,\beta_x,w_0} = x(x-a)$

  at $y = 0$ or b, $\quad u_0 = \beta_y = w_0 = 0 \;\Rightarrow f_{dist}^{u_0,\beta_y,w_0} = y(y-b)$
- Load: **Sinusoidally distributed** ($q_0 = 20$)

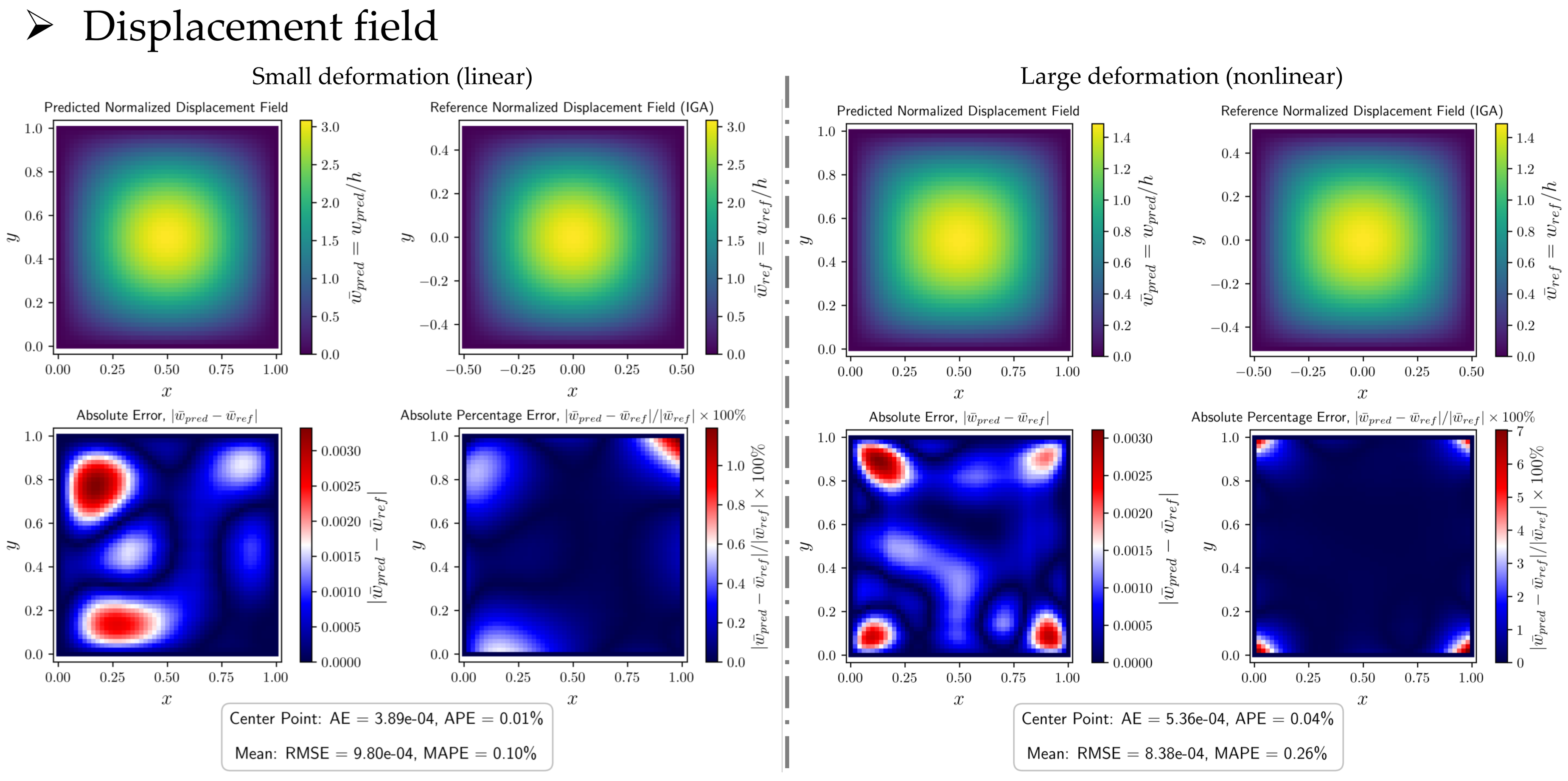
$$q(x,y) = q_0\, E\left(\frac{h}{a}\right)^4 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

- Device: **NVIDIA RTX 4090 Laptop GPU 16GB VRAM**
- PINN model: **ANN(2, 32, 'gelu', 32, 'gelu', 5)**
- Optimizer: **Limited-memory Broyden–Fletcher–Goldfarb–Shanno (L-BFGS)**
- Random sampling: **Sobol method**
- Numerical integration: **Node-based Delaunay method**



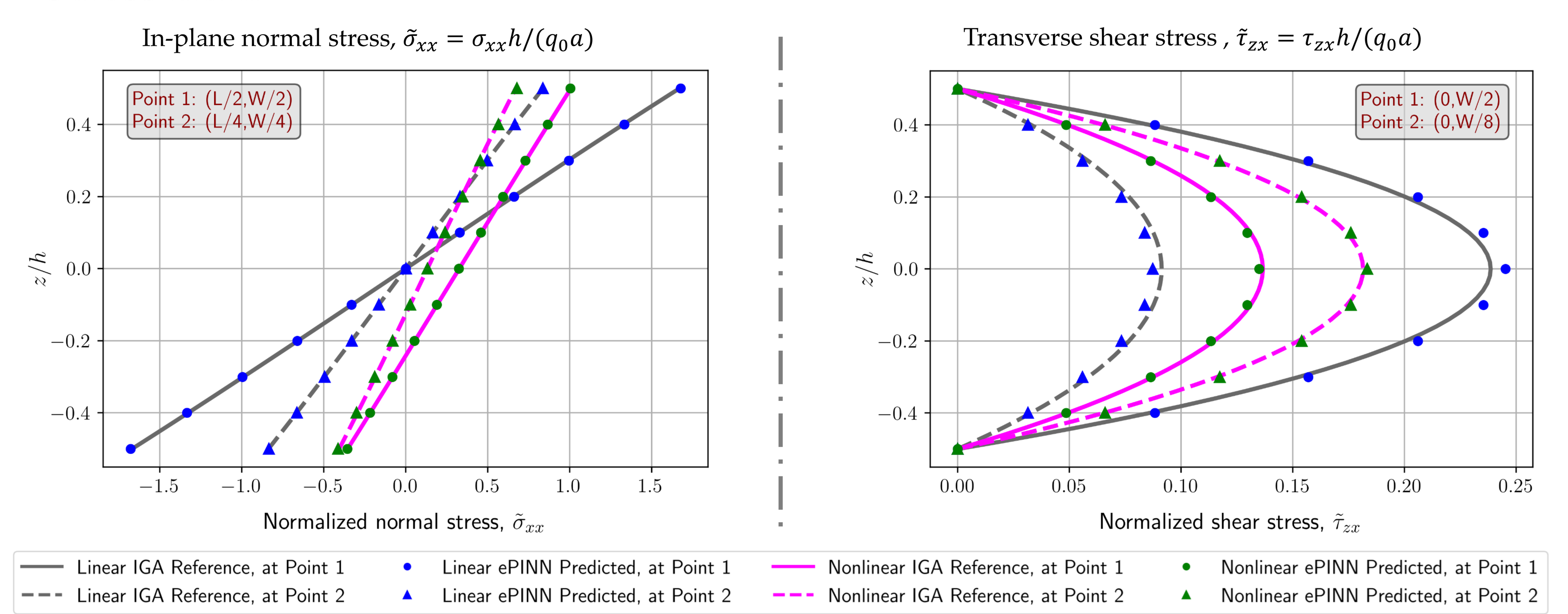


➢ Displacement field

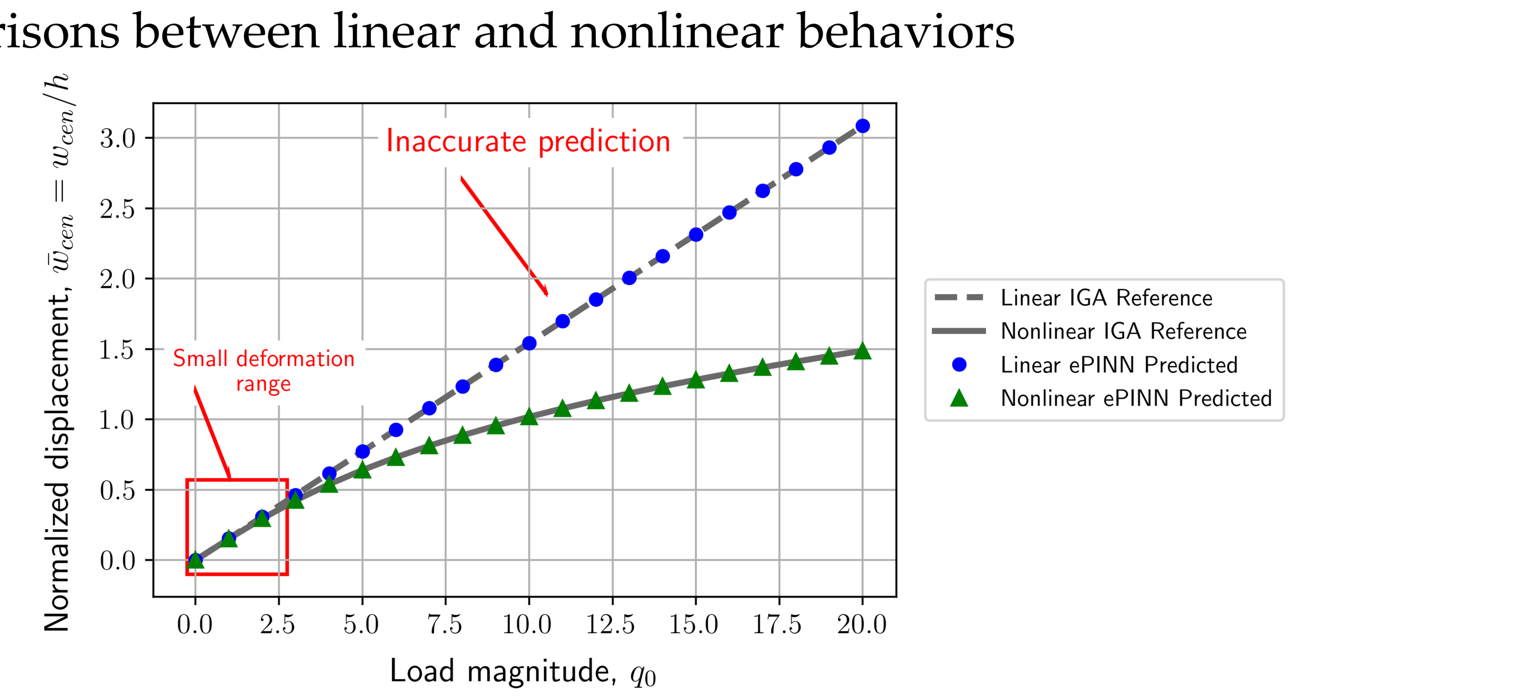


* IGA (Isogeometric analysis)

➢ Stress field



➢ Comparisons between linear and nonlinear behaviors



Conclusions

➢ Successfully integrated ePINNs with HSDT, eliminating mesh discretization and iterative solvers for complex, geometrically nonlinear bending of TPMS plates.

➢ Effectively enforced hard-coded boundary conditions via specialized distance functions, eliminating the need for penalty losses, ensuring exact physical adherence.

➢ Efficiently deliver high-fidelity displacement and stress predictions on standard laptop GPU hardware.